Modeling Energy Demand Aggregators for Residential Consumers

Giuseppe Di Bella, Laura Giarré, Mariano Ippolito, Alain Jean-Marie, Giovanni Neglia, Ilenia Tinnirello

Modeling Energy Demand Aggregators for Residential Consumers

Giuseppe Di Bella, Laura Giarrè , Mariano Ippolito,
Alain Jean-Marie, Giovanni Neglia, Ilenia Tinnirello

RESEARCH
REPORT
N° 8355
September 2013
Project-Teams Maestro

ISSN 0249-6399 ISRN INRIA/RR--8355--FR+ENG

Modeling Energy Demand Aggregators for Residential Consumers

Giuseppe Di Bella*, Laura Giarrè *, Mariano Ippolito*,
Alain Jean-Marie†‡, Giovanni Neglia†, Ilenia Tinnirello*

Project-Teams Maestro

Research Report n° 8355 — September 2013 — 18 pages

Abstract: Energy demand aggregators are new actors in the energy scenario: they gather a group of energy consumers and implement a demand-response paradigm. When the energy provider needs to reduce the current energy demand on the grid, it can pay the energy demand aggregator to reduce the load by turning off some of its consumers' loads or postponing their activation. Currently this operation involves only greedy energy consumers like industrial plants. In this report we want to study the potential of aggregating a large number of small energy consumers like home users as it may happen in smart grids. In particular we want to address the feasibility of such approach by considering which scale the aggregator should reach in order to be able to control a significant power load. The challenge of our study derives from residential users' demand being much less predictable than that of industrial plants. For this reason we resort to queuing theory to study analytically the problem and quantify the trade-off between load control and tolerable service delays.

Key-words: energy demand aggregators, queueing theory, Poisson process

A shorter version of this technical report is going to apper in the proceeding of the 52nd IEEE Conference on Decision and Control.

* Dipartimento di Energia, Ingegneria dell'informazione e Modelli Matematici, Università di Palermo, Italy.
† Inria, Centre de Recherche de Sophia-Antipolis Méditerranée, Equipe-Projet MAESTRO
‡ LIRMM, CNRS/Université Montpellier 2

RESEARCH CENTRE
SOPHIA ANTIPOLIS – MÉDITERRANÉE

2004 route des Lucioles - BP 93
06902 Sophia Antipolis Cedex

Modélisation des agrégateurs de consommation d'énergie pour le secteur résidentiel

Résumé : Les agrégateurs de consommation d'énergie sont de nouveaux acteurs dans le scénario énergétique : ils rassemblent un groupe de consommateurs d'énergie et ils mettent en œuvre un paradigme demande-réponse. Lorsque le fournisseur d'énergie a besoin de réduire la consommation énergétique actuelle sur la grille, il peut payer l'agrégateur pour réduire la charge en désactivant la requête de certains de ses consommateurs ou en retardant leur activation. Actuellement, cette opération ne concerne que des consommateurs gourmands comme les installations industrielles. Dans ce rapport, nous voulons étudier la possibilité de regrouper un grand nombre de petits consommateurs d'énergie, comme les utilisateurs du secteur résidentiel qui peuvent être connectés par une *smart grid*. En particulier, nous voulons aborder la faisabilité d'une telle approche en considérant quelle échelle l'agrégateur devrait atteindre afin d'être en mesure de contrôler une charge de puissance importante. Le défi de notre étude provient de la demande des utilisateurs résidentiels qui est beaucoup moins prévisible que celle des installations industrielles. Pour cette raison, nous avons recours à la théorie des files d'attente pour étudier analytiquement le probléme et quantifier le compromis entre contrôle de la charge et retards de service acceptables.

Mots-clés : agrégateurs de consommation d'énergie, théorie des files d'attente, processus de Poisson

1 Introduction

The current world-wide increase of energy demand cannot be matched by energy production and power grid update. In particular more and more often the power grid is not able to satisfy the peak demand (as the recent big blackouts in USA or in Europe revealed). For this reason, one of the solutions currently evaluated is to shift energy demand to the moment of the day when it can be satisfied more easily. The increasing use of renewable energies makes this approach even more interesting, given the high time-variability and unpredictability of sun light or wind intensity.

A new figure is appearing in the energy market: the "energy demand aggregator," that gathers a group of energy consumers. When the energy provider needs to reduce the current energy demand on the grid, it can pay the energy demand aggregator to reduce the load by turning off some of its consumers loads (and turning them on later). For example EnergyPool [1] can control up to 1GW power demand.

At the moment this approach is limited to energy-greedy users like industries, but also some residential appliances may be activated with some flexibility, as it is the case of water heaters, dishwashers or laundry machines. To the best of our knowledge we are the first to evaluate quantitatively the performance achievable by aggregating a large number of home users. By our analysis we aim to answer the fundamental question of which aggregation scale should be reached in order to have a significant power peak reduction with an acceptable delay experienced by each user. Our analysis is also of interest for smart grids where a central controller may manage all the appliances connected to the local grid. For the sake of simplicity we will simply refer to an aggregator in the rest of the report.

We assume the following operation for a typical demand-response system. The energy supplier communicates with an adequate advance to the energy aggregator its demand expressed by a cap K on the maximum absorbed power to be enforced during a specific time interval $[T_s, T_e]$. We assume that the aggregator has the information on the users' instantaneous power consumption and control the plugs at each user's home, but the appliances do not have any particular intelligence. The aggregator can then enforce the supplier's demand only by disconnecting a subset of the plugs and then postponing the load of the corresponding appliances but cannot anticipate it. In case the appliances already working cannot be disconnected for efficiency reasons, the aggregator can opportunistically anticipate the starting of the load control in order to guarantee that the supplier's demand is satisfied in the desired time interval. The continuous monitoring allows the aggregator to reconnect some plugs if the instantaneous power demand is below the cap during the controlled period.

Under this form of control users may wait longer for their appliances to complete their task because of the time during which the plug is disconnected (but they will share a part of the aggregator's revenues). This additional delay is a random variable (because it depends on the time instant the user would have liked to turn on his/her appliance) whose average is an increasing function of the control time duration $T_e - T_s$ and a decreasing function of the power cap. For these reasons, from the methodological point of view, we resort to queuing theory to evaluate the performance of our control. In particular, in this report, we are able to quantify this delay for any possible pair $(c, [T_s, T_e])$ and then to characterize the tradeoff between achievable power peak reduction and user's quality of service. We believe that, even under this simplified model, our approach is useful to understand which size an aggregator should reach (i.e. how many users it should coalesce) in order to be able to control a power demand significant for the supplier without a significant service quality degradation for the user. As we said above, we assume the aggregator is not able to anticipate the appliance's energy demand, but can only block it by disconnecting the plug. Moreover, we consider that active appliances cannot be disconnected and therefore the supplier's demand can be satisfied only by anticipating the starting of the load control. For these reasons, from the methodological point of view, we resort to queuing theory to evaluate the performance of our control.

The rest of this report is organized as follows. After a brief literature review presented in Section 2, in Section 3 we discuss our load model and a numerical example of load profiles referring to a typical flexible appliance (namely, the laundry machine). In Section 4 we model the effect of the aggregator control on the number of active appliances during the control period and quantify the service delay distribution. In Section 5 we present some numerical results enlightening the tradeoffs between power reduction, quality of service and number of users controlled by the aggregator. Finally, our conclusions are discussed in Section 6.

2 Literary Review

The problem of energy demand aggregation has been recently studied, and the importance of exchanging information among end users and energy producers has been investigated to underline, model or control different related aspects. In [2],[3], the energy demand aggregation when industrial plants are integrated in the electricity network are studied, while [4] analyzes the economic effects of aggregation in residential areas. Here the model synthesizes a daily load profile based on load profiles of Dutch residential customers. Simulated data representing aggregate demands of domestic appliances and electric vehicles are presented and used. The load profiles are based on [5], where the energy demand is modeled by using Monte Carlo simulations, and normalized aggregate load profiles are provided for electric vehicles and four typical domestic appliances, divided in *wet* appliances (laundry machines and dishes machines) and *cold* appliances (refrigerators and freezers).

The effects of the aggregation and how it will affect the energy market is analyzed in [6] via a game theoretical approach, while the overall complexity of enabling reliable electricity service in the changing industry is studied in [7]. Load management has been studied and adaptive solutions have also been provided in literature, such as in [8] where a multi-layered adaptive load management system is studied to integrate large scale demand-response features efficiently and reliably. Moreover, in competitive power markets, with increasing penetration of variable renewable energy resources such as wind power, it becomes more challenging for energy demand aggregators to manage their electricity cost because of the presence of further uncertainties, as shown in [9]. In [10] different types of aggregator nodes are organized hierarchically in a tree (called the Smart Link) and can cooperatively create highly adaptable load control strategies to meet a given load reduction target. Optimal management of consumer flexibility in an electric distribution system is studied also in the EU project ADDRESS [11], where, as in [12], the aggregation of a number of consumers clustered according to appropriate criteria, is presented as one of the most promising approaches for modifying the daily load profile at nodes of an electric distribution network.

3 Load Model

We consider the power consumption originated by the aggregation of *one type only* of appliances (whose activation time can be flexible for the users). Since each user consumption coincides with its appliance consumption, in the following we indifferently refer to the total number of appliances or users.

Some statistical studies [13, 5] have characterized the percentage of users activating a specific residential appliance along different intervals of the day. In these studies, the day is divided into

Appliance	6	8	10	12	14	16	18	20	22	24
Dishwasher.	3	9	9	3	13	0	16	38	13	3
Laundry m.	16	28	38	19	16	19	16	16	3	6

Table 1: Examples of appliance activation rates [% over 30000 users]

equal size intervals and the percentage of active users is averaged in each interval. Assuming that the user population U is large enough and considering an observation time of one day, we can model the activation of a new appliance as a non-homogeneous Poisson process with arrival rate $\lambda(t)$. The working interval of the reference appliance is usually deterministic or a deterministic function of the activation program. Let D be the time interval during which the appliance keeps working after its activation. We assume that all the considered appliances are homogeneous, with the same working interval, and power consumption.

Let $u(t)$ be the total number of users whose appliance is on at time t. Since in absence of critical problems the energy production is able to follow exactly the energy demand we can model the $u(t)$ random process as the number of jobs in a $M(t)/D/\infty$ queue.[1] We can easily characterize the probability distribution $p_i(t) = Pr\{u(t) = i\}$ to find i active users at time t. Since all the appliances activated before $t - D$ are deterministically switched off in t, we find i users at time t if exactly i new appliances have been switched on during the last D interval:

$$p_j(t) = \frac{[\bar{\lambda}_D^{t-D} D]^j}{j!} e^{-\bar{\lambda}_D^{t-D} D} \tag{1}$$

where $\bar{\lambda}_D^{t-D} = 1/D \int_{t-D}^{t} \lambda(x)dx$ is the average arrival rate in a D interval starting in $t - D$. We observe that there is no correlation between the number of active users in t and $t+D$. Conversely, we can derive the probability distribution $p_j(t+\Delta t|u)$ of j active users at $t+\Delta t$ for $\Delta t < D$ given that there are u active users in t, by considering the joint probability to have k departures and $j-u+k$ arrivals in $[t, t+\Delta t]$. The probability $a(t, \Delta t)$ that an appliance active in t is switched off by $t+\Delta t$ can be expressed as the probability that its arrival has occurred by $t-D+\Delta t$, i.e.:

$$a(t, \Delta t) = \frac{\int_{t-D}^{t-D+\Delta t} \lambda(x)dx}{\int_{t-D}^{t} \lambda(x)dx}. \tag{2}$$

Eq. 2 follows from the definition of conditional probability. Therefore, the probability $d_k(t, \Delta t|u)$ that k appliances among the u active at time t are switched off during $[t, t+\Delta t]$ is expressed by the binomial:

$$d_k(t, \Delta t|u) = \binom{u}{k} a(t, \Delta t)^k (1 - a(t, \Delta t))^{u-k},$$

for $k = 0, 1, \ldots u$. Finally, being $l = \max\{0, u - j\}$, the conditioned probability to have j active users can be expressed as:

$$p_j(t + \Delta t|u) = e^{-\bar{\lambda}_{\Delta t}^{t} \Delta t} \sum_{k=l}^{u} d_k(t, \Delta t|u) \frac{(\bar{\lambda}_{\Delta t}^{t} \Delta t)^{j-u+k}}{(j-u+k)!} \tag{3}$$

In the assumption that the aggregator can monitor the actual number of active users at the current time instant t, equation 3 allows to evaluate the distribution of the number of active users at $t + \Delta t$.

[1] Here "M(t)" (for "Markovian") denotes the Poisson time-varying appliance activation process, "D" indicate that the operation time is deterministic and ∞ the presence of an infinite number of servers so that new requests can immediately be served. The reader can refer to [14] for a basic introduction to queuing theory.

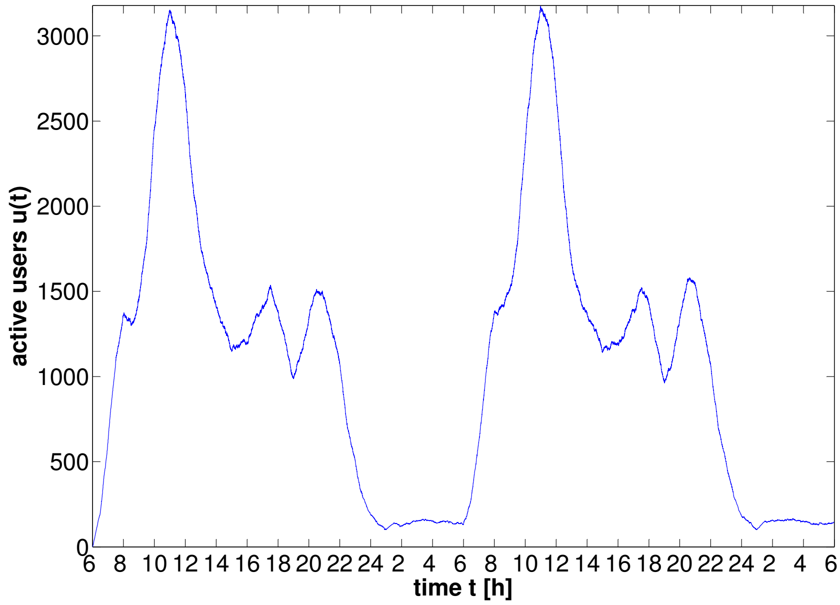


Figure 1: A random realization of the number of active appliances for a population of 10000 total users

3.1 Numerical Example

We suppose that the load aggregator knows the statistical data about the usage of residential appliances along the day. Consider, for example, two typical appliances with flexible activation intervals such as the dishwasher and the laundry machine. Their activation rate is shown in table I [13] as a percentage of active appliances over a total number of 30000 potential appliances. Although the data are available in intervals of 30 minutes, for sake of readability, the rate is averaged in intervals of two hours starting from the time specified in the first row, and the last three intervals from midnight to 6a.m have been accumulated in a single value. For the laundry machine data, assuming D equal to 90 minutes, figure 1 shows a realization of the random process $u(t)$ with a total population of 10000 users. From the figure we can clearly identify the peak hours and the effect of the time-varying activation rates. The maximum number of users into the system is about 3100, which corresponds to a load of 4.65 MW if we consider a power consumption equal to 1.5 KW for each laundry machine. At noon, being $\bar{\lambda}_{1.30}^{10.30} = 0.38 \cdot 10000/2h = 1900$ arrivals/h, the average number of users is $\bar{\lambda}_{1.30}^{10.30} D = 2850$.

In figure 2 we also plotted the probability distribution $p_i(t)$ of active users at different hours of the day (lines) and the corresponding numerical distributions (points) evaluated through Monte Carlo simulations with 10^5 samples.

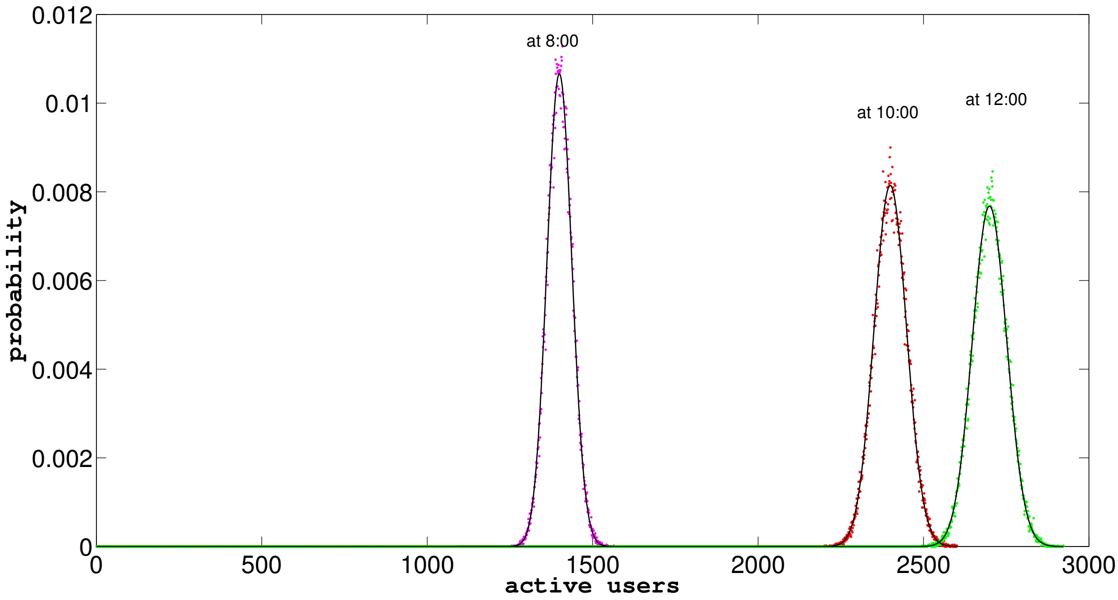


Figure 2: Probability distribution $p_i(t)$ of the number active users in different hours of the day: analytical (lines) and numerical (points) results.

4 Load Model under Aggregation Control

Consider now the effect of the aggregator on the number of active users. Let c be the maximum number of appliances that can be simultaneously active during the control period. We assume that the load controller can only work on the new activation requests, because disconnecting appliances whose working cycle is in progress can be inefficient and uncomfortable for the users. In order to guarantee that less than c appliances are active in the desired time interval $[T_s, T_e]$, the load aggregator anticipates the starting of the load control to $T_{sc} = T_s - D$, because the appliances active in T_{sc} (whose number can be higher than c) will have necessarily terminated their operation by T_s.

During the control period $[T_{sc}, T_e]$, each appliance activation is conditioned to an admission control: the appliance can be switched on only if the number of appliances already active is lower than c. When the new appliance cannot be activated, the aggregator disconnects the relative plug until some power resources become available. We assume that there is no limit to the maximum number of disconnected appliances and that they can be orderly reconnected into the system (when possible) according to the arrival time of their activation request. Under these assumptions, we can model the load process with aggregation control as an $M(t)/D/c$ process.

4.1 Active and disconnected appliances

Let $u(t)$ be the number of appliances at time t that are working or are waiting for some available power in order to start (their plug has been disconnected by the aggregator). The probability distribution $p_j(T_{sc})$ of $u(T_{sc})$ can be evaluated on the basis of the previous analysis without admission control. Consider now a generic time interval $\Delta t < D$. The number of users $u(T_{sc}+\Delta t)$ in the system can be obtained by $u(T_{sc})$ considering the difference between the number of new activation requests and the number of appliances that have been switched off in the interval

$[t, t+\Delta t]$. While the new requests depend on the average arrival rate $\bar{\lambda}_{\Delta t}^{T_{sc}}$, the number of appliances switched off depends on the number of arrivals in the previous $[T_{sc}-D, T_{sc}-D+\Delta t]$ interval, being the total number of arrivals in the interval $[T_{sc}-D, T_{sc}]$ equal to $u(T_{sc})$. In other words, $p_j(T_{sc}+\Delta t|u(T_{sc}))$ can be expressed as equation 3. By weighting each conditioned probability with the probability of the conditioning event to have $u(T_{sc})$ users in T_{sc}, we can find the probability to have j users in the system at time $T_{sc}+\Delta t$:

$$p_j(T_{sc}+\Delta t) = \sum_{u=0}^{\infty} p_j(T_{sc}+\Delta t|u)p_u(T_{sc}). \tag{4}$$

Consider now a generic instant $t > T_{sc}+D$. The number of appliances $u(t)$ is now depending on $u(t-D)$, because the appliances disconnected in $t-D$ are still into the system at time t. Indeed, $u(t)$ is given by the sum of $u(t-D)-c$ (if positive) and the new arrivals, i.e. we can find j appliances in t if we have $j+\min\{0, c-u(t-D)\}$ new arrivals:

$$p_j(t) = \sum_{i=0}^{j+c} p_i(t-D)\frac{[\bar{\lambda}_D^{t-D}D]^{j+\min\{0,c-i\}}}{(j+\min\{0,c-i\})!}e^{-\bar{\lambda}_D^{t-D}D} \tag{5}$$

Since equation 4 allows to know $p_i(t)$ for $t \in [T_{sc}, T_{sc}+D]$, for $t \in [T_{sc}+D, T_e]$ it is possible to apply equation 5 in $k = \lfloor (t-T_{sc})/D \rfloor$ consecutive time intervals starting from the the distribution $p_i(T_{sc}+(t-T_{sc})\%D)$. Therefore, the behavior of the user population can be characterized during the whole control period $[T_{sc}, T_e]$. The probability distributions $q_j(t)$ to have j disconnected appliances is obviously $p_{j+c}(t)$ for $j > 0$ and $\sum_{i=0}^{c} p_i(t)$ for $j = 0$.

4.2 Delay analysis

During the control period, some appliances cannot be activated exactly when the user makes the activation request in order to guarantee that the total power consumption of the system is bounded to a desired value. In this case, they experience a *service delay* until some power resources become available. Consider a generic appliance whose activation request is originated in $t \in [T_{sc}, T_e]$. Let $E[W(t)]$ be its average service delay. Since the arrival rate of activation requests in t is given by $\lambda(t)$, the average delay experienced by a random user when the load control is applied can be expressed as:

$$\bar{W}_{T_e-T_{sc}}^{T_{sc}} = \frac{\int_{T_{sc}}^{T_e} \lambda(t)E[W(t)]dt}{\int_{T_{sc}}^{T_e} \lambda(t)dt}. \tag{6}$$

Eq. 6 is proven in Appendix A.

To derive $E[W(t)]$ we evaluate the cumulative distribution of the delay $W(t)$ experienced by a user arriving in t following the derivation proposed in [15], [16]. We generalize the approach considering non-homogeneous arrival rates and an unknown initial state $u(T_{sc})$.

Let t be the arrival instant of an appliance and $u(t) = kc+i$ with $i \in [1, c]$. We imagine the $u(t)$ appliances to be ranked according to their arrival order. The appliance arrived at t has to wait that $(k-1)c+i$ appliances complete their service before being reconnected (i.e. before being in the first c positions). Since only c appliances can complete their work in an interval equal to D, the new appliance is going to be reconnected in the interval $[t+(k-1)D, t+kD]$. Consider for example the scenario depicted in figure 3. The new arrival in t (the yellow one in the figure) is in position 7 in a system in which $c = 2$. Since it has to wait that 5 users complete their service, it will be activated after $t+2D$ and before $t+3D$. Specifically, its delay will be

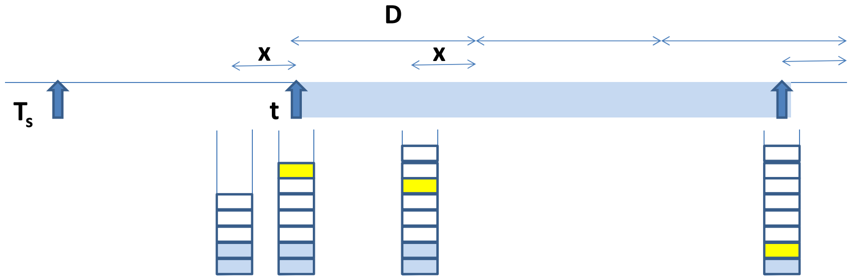


Figure 3: A possible arrival and queue scenario (the yellow is the new arrival)

lower than $3D - x$ with $x \in [0, D]$, if the target appliance occupies one of the first c positions of the queue by $t + 3D - x$. If we go backward in the past, this condition implies that the target appliance has to be in one of the first $2c$ positions by $t + 2D - x$, and in one of the first $3c$ positions by $t + D - x$. To satisfy this last condition, the number of users in queue at $t - x$ plus the number of new arrivals before t has to be strictly lower than $3c$ (note that the number of active appliances at $t - x$ are switched off by $t - x + D$ and therefore are not in the system anymore).

We can generalize the previous considerations for evaluating the cumulative probability $P\{W(t) \leq kD - x\}$ as the probability that the number of users $N_t(t + D - x)$ arrived before t that are still into the system at $t + D - x$ is strictly lower than kc. For a given value of the queue p in $t - x$, $N_t(t + D - x)$ is lower than kc if the number of new arrivals is strictly lower than $kc - p$. Therefore, $P\{W(t) \leq kD - x\}$ is given by:

$$\sum_{p=0}^{\infty} q_p(t-x) \sum_{j=0}^{kc-p-1} \frac{(\bar{\lambda}_x^{t-x} x)^j}{j!} e^{\bar{\lambda}_x^{t-x} x} \tag{7}$$

Obviously, for $t - x < T_{sc}$ (i.e. before the starting of the admission control), $q_0(t - x)$ is equal to 1 (there is no disconnected user in the system).

5 Numerical Results

In order to answer to our initial problem about the scale of the energy demand aggregator, we quantify the service delay experienced by the users for different control actions $(c, [T_s, T_e])$ and for different user populations.

We assume that the total user population U represents the number of users served by the same primary substation of the power grid that subscribe the aggregation control. These users accept that the activation of their appliances can be delayed for responding the power demand reduction requested by the energy supplier and in returns share the aggregator revenues. In particular, we focus on the control of a given appliance type that in our experiments is the laundry machine. The activation rate of the laundry machine follows the profile summarized in 3.

We assume that the energy supplier asks a power reduction K to be applied starting from 10.00 a.m. (during the peak hours). The desired power reduction is computed by considering the average value or a given percentile of the expected power demand. Indeed, the energy supplier can derive the probability distribution of the power demand on the basis of the power consumption data monitored during the previous day or a recent time window.

The desired power reduction K to be applied to the average value (or percentile) of the power demand is mapped into the maximum number c of new appliances that can be simultaneously active in $[T_s, T_e]$.

Consider first the case of the maximum size of user population, i.e. all the primary substation users (typically, tens of thousands) join the enery aggregator controller. Figure 4 shows the average service delay experienced when the power reduction K (in the range 100 KW-5MW) is applied starting from 8.30 a.m. (i.e. 10.00 a.m. - 90 min) to a total population of 30000 users. The power reduction demand is expressed considering the average value (left plot) and the 90% percentile (right plot) of the power demand. The first case corresponds to a smaller value of c. From the figure we can see that in the maximum considered control period (namely, 180 min), asking for a power reduction of 0.5 MW from 10.00 a.m. to 13.00 a.m. leads to a service delay lower than 20 min. For higher power reduction values, the service delay can be still acceptable if the control interval is limited. By comparing figures 4-a and 4-b, we can also observe that there is not a significative difference between applying the power reduction to the 90% percentile of the power demand rather or to the average value. This is due to the fact that we are considering a large user population which corresponds to a small variability of the aggregated power demand. The projection of the curves in figure 4 is also visualized in figure 6-a.

Figure 5 shows some curves (analogous to the ones visualized in figure 4) for different user populations. As the number of aggregated users increases, for a given power reduction, the system is obviously able to provide a lower service delay. For example, for a power reduction of 100 KW and a control interval of 3 hours, the average service delay is lower than 20 minutes when $U = 10000$ and about 70 minutes when $U = 3000$. For small user populations, the service delay experienced when the power reduction refers to the 90% percentile of the power demand is significantly smaller than when it refers to the average value (e.g. about 10 minutes of reduction for $U = 3000$, $K = 100KW$ and $T_e - T_s$=3 hours).

The projection of the curves in 5-a is visualized in 6-b, for the cases $U = 10000$ and $U = 3000$. The figure clearly shows the tradeoff between the power reduction and the control period. For example, for $U = 3000$ an average service delay of about 40 min is experienced asking for a power reduction of 200 KW for 180 min, or for a power reduction of 350 KW for 60 minutes. If we consider these values good estimates for the maximum delay that users would tolerate and for the minimum power reduction demanded from the energy supplier (at the level of a primary substation), we conclude that the aggregator should be able to coalesce roughly 10% of the residential users in a given area (3000 out of 30000). The control of other appliances would clearly permit to reduce such percentage and make this kind of solutions more viable.

6 Final Remarks

The main contribution of this report is proposing a queue model for characterizing the behavior of an energy demand aggregator working on homogenous appliances. The proposed model, supported by numerical simulations, leads us to answer the fundamental question of which aggregation scale should be reached in order to have a significant power peak reduction with an acceptable delay experienced by each user.

We are currently working on different model extensions for taking into account heterogeneous appliances and alternative admission mechanisms to be performed by the aggregator. In the first case, the extension is straightforward if the appliances only differ for the heterogeneous load profile but have the same power consumption. In the second case, we consider the possibility to monitor the instantaneous power consumption only at regular time instants. The admission control can be performed by using the model for estimating the aggregated power consumption during the time interval between consecutive power readings.

References

[1] O. Baud, "Energy-pool," Mar. 2013. http://www.energy-pool.eu/index.php?lang=en.

[2] C. Ramirez-Escobar, C. Alvarez-Bel, and N. Georgantzis, "Controlling market power of vertically integrated firms in electricity networks: Demand response of aggregator agents," in *Innovative Smart Grid Technologies (ISGT Latin America), 2011 IEEE PES Conference on*, pp. 1–7, Oct.

[3] O. Abdalla, M. Bahgat, A. Serag, and M. El-Sharkawi, "Dynamic load modelling and aggregation in power system simulation studies," in *Power System Conference, 2008. MEPCON 2008. 12th International Middle-East*, pp. 270 –276, march 2008.

[4] A. Abdisalaam, I. Lampropoulos, J. Frunt, G. Verbong, and W. Kling, "Assessing the economic benefits of flexible residential load participation in the dutch day-ahead auction and balancing market," in *European Energy Market (EEM), 2012 9th International Conference on the*, pp. 1–8, May.

[5] S. Pagliuca, I. Lampropoulos, M. Bonicolini, B. Rawn, M. Gibescu, and W. L. Kling, "Capacity assessment of residential demand response mechanisms," in *Universities' Power Engineering Conference (UPEC), Proceedings of 2011 46th International*, pp. 1–6, Sept.

[6] C. Chen, S. Kishore, Z. Wang, M. Alizadeh, and A. Scaglione, "How will demand response aggregators affect electricity markets? a cournot game analysis," in *Communications Control and Signal Processing (ISCCSP), 2012 5th International Symposium on*, pp. 1–6, May.

[7] M. Ilic, "Evolution of reliability toward meeting grid-enabled users needs," in *Power and Energy Society General Meeting, 2011 IEEE*, pp. 1–5, July.

[8] J.-Y. Joo and M. Ilic, "A multi-layered adaptive load management (alm) system: Information exchange between market participants for efficient and reliable energy use," in *Transmission and Distribution Conference and Exposition, 2010 IEEE PES*, pp. 1–7, April.

[9] Y. Xu, L. Xie, and C. Singh, "Optimal scheduling and operation of load aggregators with electric energy storage facing price and demand uncertainties," in *North American Power Symposium (NAPS), 2011*, pp. 1–7, Aug.

[10] P. Carreira, R. Nunes, and V. Amaral, "Smartlink: A hierarchical approach for connecting smart buildings to smart grids," in *Electrical Power Quality and Utilisation (EPQU), 2011 11th International Conference on*, pp. 1–6, Oct.

[11] ADDRESS, "European project," Mar. 2013. http://www.addressfp7.org.

[12] A. Agnetis, G. Dellino, G. De Pascale, G. Innocenti, M. Pranzo, and A. Vicino, "Optimization models for consumer flexibility aggregation in smart grids: The address approach," in *Smart Grid Modeling and Simulation (SGMS), 2011 IEEE First International Workshop on*, pp. 96–101, Oct.

[13] R. Miceli, "Sustainable development and energy saving laboratory," tech. rep., DIEET - University of Palermo, August 2007.

[14] L. Kleinrock, *Queueing Systems. Volume 1: Theory.* wiley, 1975.

[15] G. Franx, "A Simple Solution for the M/D/c Waiting Time Distribution," *Operation Research Letters*, vol. 29, no. 5, pp. 221–229, 2001.

[16] G. Franx, "The transient m/d/c queueing system," 2002.

[17] E. Çinlar, *Introduction to Stochastic Processes.* New Jersey: Prentice-Hall, 1975.

A Proof of Equation 6

We consider a non-homogeneous Poisson arrival process $\mathcal{N}(t), t \geq 0$ with a continuous intensity rate $\lambda(t)$ and expectation function $\Lambda(t) = \int_0^t \lambda(x)\mathrm{d}x$. $\{\tau_1, \tau_2, ...\tau_n, \dots\}$ denotes the sequence of arrivals for $t \geq 0$. Let W_{τ_i} be a stochastic process defined on the points of $\mathcal{N}(t)$.

We prove that:

$$\mathrm{E}\left[\frac{\sum_{i|\tau_i<T} W_{\tau_i}}{\mathcal{N}(T)}\right] = \frac{\int_0^T \lambda(t)\mathrm{E}[W_t]\mathrm{d}t}{\int_0^T \lambda(t)\mathrm{d}t} = \frac{\int_0^T \lambda(t)\mathrm{E}[W_t]\mathrm{d}t}{\Lambda(T)}, \tag{8}$$

where $E[W_t]$ indicates the expected value of the random variable W_t and it is then conditioned on the fact that there is an arrival in t. In particular, we are going to prove that:

$$\mathrm{E}\left[\sum_{i=1}^{n} W_{\tau_i} | \mathcal{N}(T) = n\right] = n \, \frac{\int_0^T \lambda(t)\mathrm{E}[W_t|\mathcal{N}(t) = n, \tau_j = t \text{ for some } j = 1, \dots n]\mathrm{d}t}{\Lambda(T)}, \tag{9}$$

from which our result follows immediately by deconditioning.

We first derive the following formula for the joint probability density function of the ordered arrival times of $\mathcal{N}(t)$ in the interval $[0, T]$:

$$f_{\mathcal{N}}\left(\tau_1 = s_1, \tau_2 = s_2, \dots \tau_n = s_n | \mathcal{N}(T) = n\right) = \frac{n!}{\Lambda(T)^n} \prod_{i=1}^{n} \lambda(s_i). \tag{10}$$

Proof. Let us define $\hat{\tau}_i = \Lambda(\tau_i)$. The point process $\{\hat{\tau}_1, \hat{\tau}_2, \dots, \hat{\tau}_n, \dots\}$ is a homogeneous Poisson process $(\hat{\mathcal{N}}(t))$ with rate 1 [17], for which it is known that

$$f_{\hat{\mathcal{N}}}(\hat{\tau}_1 = u_1, \hat{\tau}_2 = u_2, \dots \hat{\tau}_n = u_n | \hat{\mathcal{N}}(t) = n) = \frac{n!}{t^n}.$$

Clearly the event $\mathcal{N}(T) = n$ corresponds to the event $\hat{\mathcal{N}}(\Lambda(T)) = n$. For a given number of arrivals n, let $(\lambda(s_i))_{ii}$ be the (diagonal) Jacobian of the transformation $u_i = \Lambda(s_i)$ for $i = 1, 2, \dots n$. Applying the change of variables formula, we obtain:

$$\begin{aligned} f_{\mathcal{N}}(s_1, s_2, \dots s_n \quad | & \quad \mathcal{N}(T) = n) = \\ = & \quad f_{\hat{\mathcal{N}}}\left(\Lambda(s_1), \Lambda(s_2), \dots \Lambda(s_n) | \hat{\mathcal{N}}(\Lambda(T)) = n\right) |(\lambda(s_i)_{ii})| = \\ = & \quad \frac{n!}{\Lambda(T)^n} \prod_{i=1}^{n} \lambda(s_i). \end{aligned}$$

□

Similarly we can calculate the joint probability density function of the arrival times without considering them in a particular order:

$$\begin{aligned}
\tilde{f}_{\mathcal{N}}\left(s_1, \dots s_n | \mathcal{N}(T)=n\right) &= \frac{1}{n!} f_{\mathcal{N}}\left(\tau_1 = s_{(1)}, \dots \tau_n = s_{(n)} | \mathcal{N}(T)=n\right) \\
&= \frac{1}{\Lambda(T)^n} \prod_{i=1}^{n} \lambda(s_i) = \prod_{i=1}^{n} \frac{\lambda(s_i)}{\Lambda(T)}, \qquad (11)
\end{aligned}$$

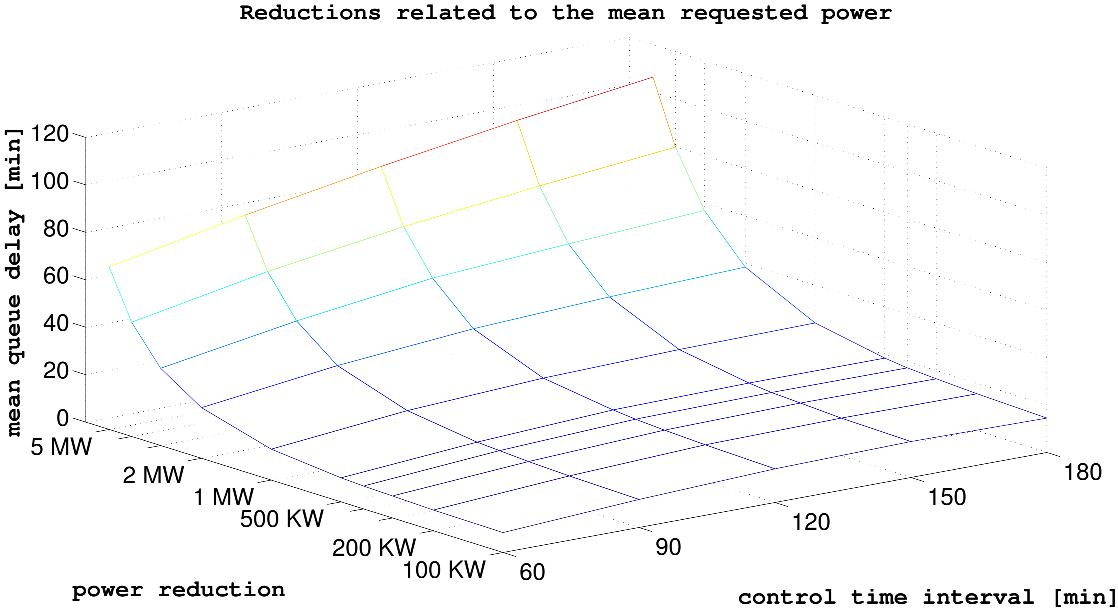
where $s_{(1)}, \dots s_{(n)}$ are the order statistics of $s_1, \dots s_n$. Let $\mathbf{s}$ and $\mathbf{s}_{-i}$ denote the vectors $(s_1, s_2, \dots s_i, \dots s_n)$ and $(s_1, s_2, \dots s_{i-1}, s_{i+1}, \dots, s_n)$ respectively. We observe that both density functions (10) and (11) are invariant under permutations of $\mathbf{s}$. We define $g_i(s_1, \dots s_n) = \mathrm{E}[W_{s_i} | \tau_1 = s_{(1)}, \dots \tau_n = s_{(n)}]$ and $g(s_1, \dots s_n) = \sum_{i=1}^{n} g_i(s_1, \dots s_n)$. $g_i()$ is invariant under permutations of $\mathbf{s}_{-i}$, while $g()$ is invariant under permutations of $\mathbf{s}$.

We are now ready to prove Eq. (9).

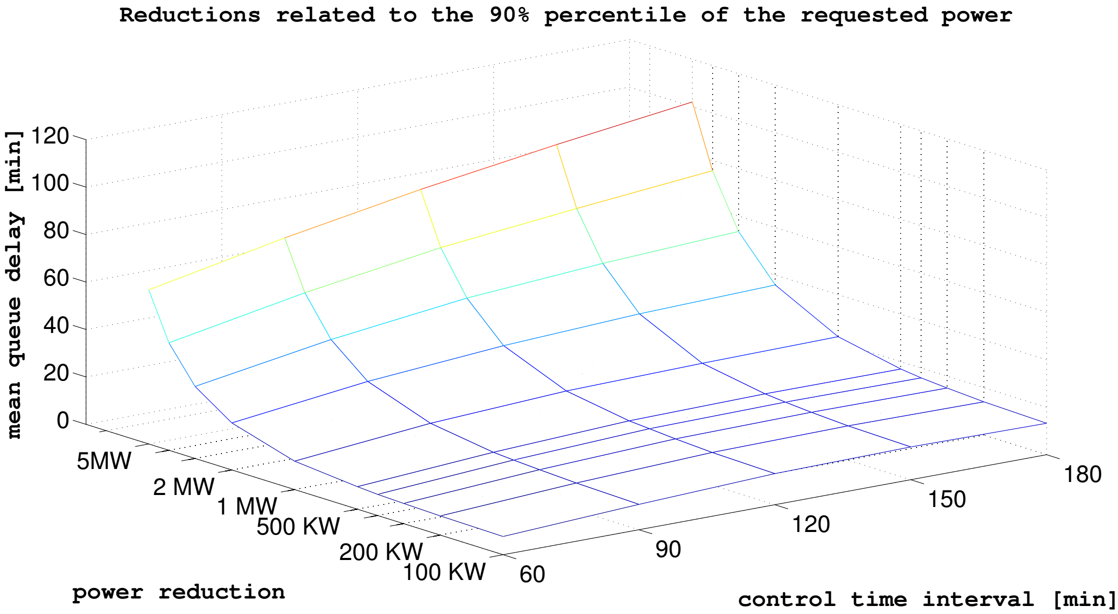
$$\begin{aligned}
& \mathrm{E}\left[\sum_{i=1}^{n} W_{\tau_i} \big| \mathcal{N}(T)=n\right] \\
&= \int_{0 \le s_1 \dots \le s_n \le T} \left(\sum_{i=1}^{n} \mathrm{E}[W_{\tau_i} | \tau_1 = s_1, \dots \tau_n = s_n]\right) f_{\mathcal{N}}\left(\mathbf{s} | \mathcal{N}(T)=n\right) \mathrm{d}\mathbf{s} \\
&= \int_{0 \le s_1 \dots \le s_n \le T} g(\mathbf{s}) f_{\mathcal{N}}\left(\mathbf{s} | \mathcal{N}(T)=n\right) \mathrm{d}\mathbf{s} \\
&= \frac{1}{n!} \int_{[0,T]^n} g(\mathbf{s}) f_{\mathcal{N}}\left(\mathbf{s} | \mathcal{N}(T)=n\right) \mathrm{d}\mathbf{s} \qquad (12) \\
&= \sum_i \frac{1}{n!} \int_{[0,T]^n} g_i(\mathbf{s}) f_{\mathcal{N}}\left(\mathbf{s} | \mathcal{N}(T)=n\right) \mathrm{d}\mathbf{s} \\
&= \sum_i \int_{[0,T]^n} g_i(\mathbf{s}) \tilde{f}_{\mathcal{N}}\left(\mathbf{s} | \mathcal{N}(T)=n\right) \mathrm{d}\mathbf{s} \\
&= \sum_i \int_{[0,T]} \left(\int_{[0,T]^{n-1}} g_i(\mathbf{s}) \tilde{f}_{\mathcal{N}}\left(\mathbf{s}_{-i} | \mathcal{N}(T)=n-1\right) \mathrm{d}\mathbf{s}_{-i}\right) \frac{\lambda(s_i)}{\Lambda(T)} \mathrm{d}s_i \qquad (13) \\
&= \sum_i \int_{[0,T]} \left(\int_{[0,T]^{n-1}} g_i(\mathbf{s}) \frac{1}{(n-1)!} f_{\mathcal{N}}\left(\mathbf{s}_{-i} | \mathcal{N}(T)=n-1\right) \mathrm{d}\mathbf{s}_{-i}\right) \frac{\lambda(s_i)}{\Lambda(T)} \mathrm{d}s_i \\
&= \sum_i \int_{[0,T]} \left(\int_{0 \le s_1 \dots \le s_{i-1} \le s_{i+1} \le \dots s_n \le T} g_i(\mathbf{s}) f_{\mathcal{N}}\left(\mathbf{s}_{-i} | \mathcal{N}(T)=n-1\right) \mathrm{d}\mathbf{s}_{-i}\right) \frac{\lambda(s_i)}{\Lambda(T)} \mathrm{d}s_i \qquad (14) \\
&= \sum_i \int_{[0,T]} \left(\int_{0 \le s_1 \dots \le s_{i-1} \le s_{i+1} \le \dots s_n \le T} \mathrm{E}[W_{s_i} | \tau_1 = s_{(1)}, \dots \tau_n = s_{(n)}]\right. \\
&\qquad \times \left. f_{\mathcal{N}}\left(\mathbf{s}_{-i} | \mathcal{N}(T)=n-1\right) \mathrm{d}\mathbf{s}_{-i}\right) \frac{\lambda(s_i)}{\Lambda(T)} \mathrm{d}s_i \\
&= \sum_i \int_{[0,T]} \mathrm{E}[W_{s_i} | \mathcal{N}(T)=n, \tau_j = s_i \text{ for some } j = 1, \dots n] \frac{\lambda(s_i)}{\Lambda(T)} \mathrm{d}s_i \\
&= n \int_{[0,T]} \mathrm{E}[W_s | \mathcal{N}(T)=n, \tau_j = s \text{ for some } j = 1, \dots n] \frac{\lambda(s)}{\Lambda(T)} \mathrm{d}s.
\end{aligned}$$

In fact (12) follows from the invariance of $g(\mathbf{s}) f_{\mathcal{N}}(\mathbf{s} | \mathcal{N}(T)=n)$ to permutations of $\mathbf{s}$, so that when we integrate on the hypercube $[0,1]^n$ we consider each ordered sequence $s_{(1)}, s_{(2)}, \dots s_{(n)}$, $n!$ times. The equality in (13) relies on the factorization of $\hat{f}()$ in (11). The invariance of $g_i(\mathbf{s}) f_{\mathcal{N}}(\mathbf{s}_{-i} | \mathcal{N}(T) = n)$ to permutations of $\mathbf{s}_{-i}$ is used in (14).

Contents

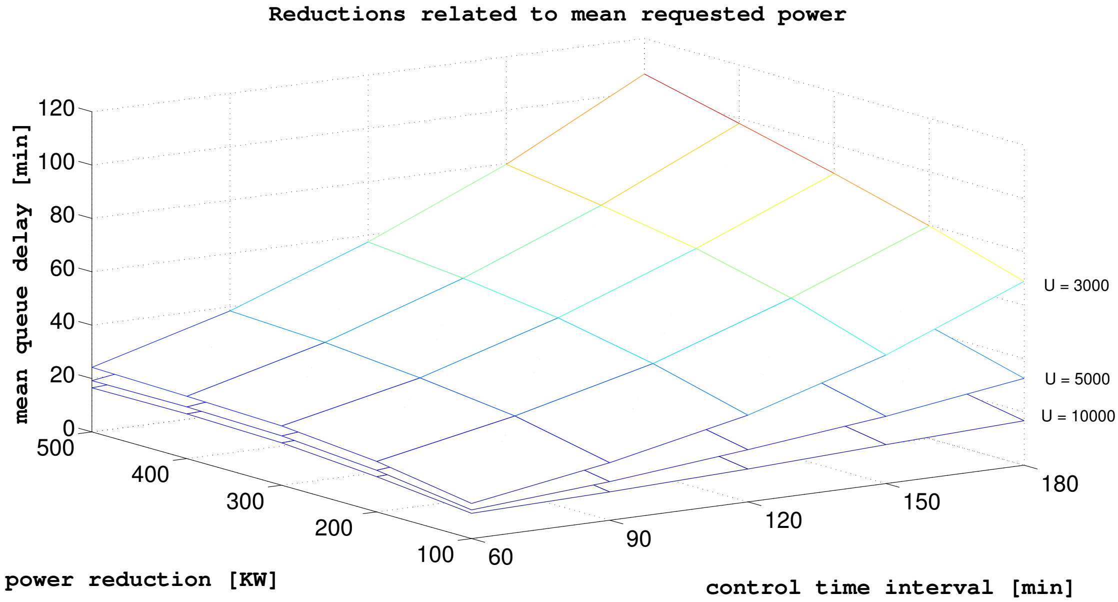


(a)

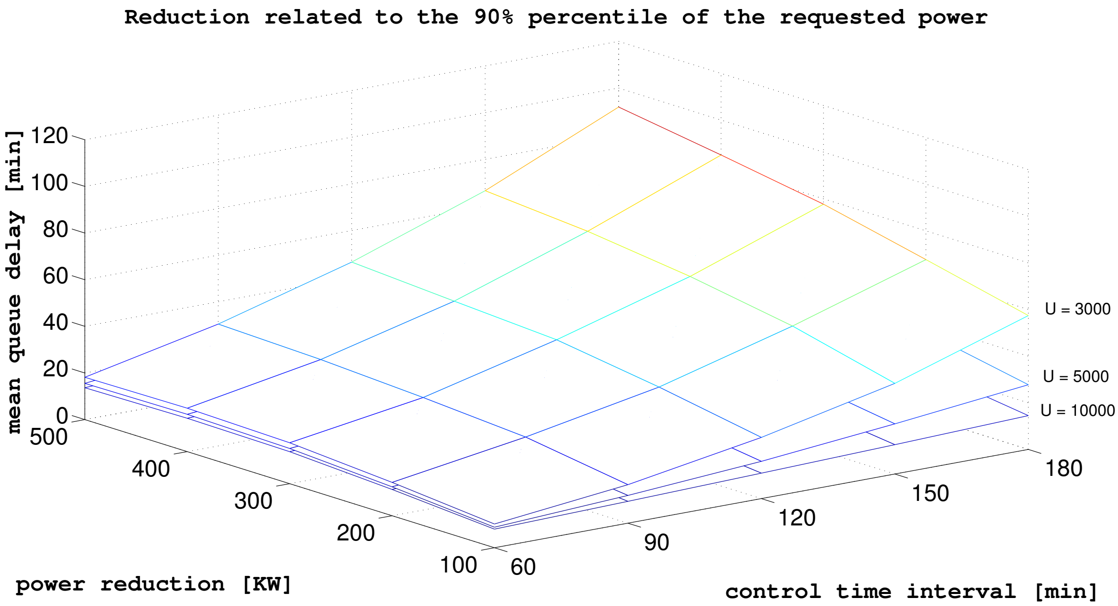


(b)

Figure 4: Average service delay for a population of 30000 laundry machines.

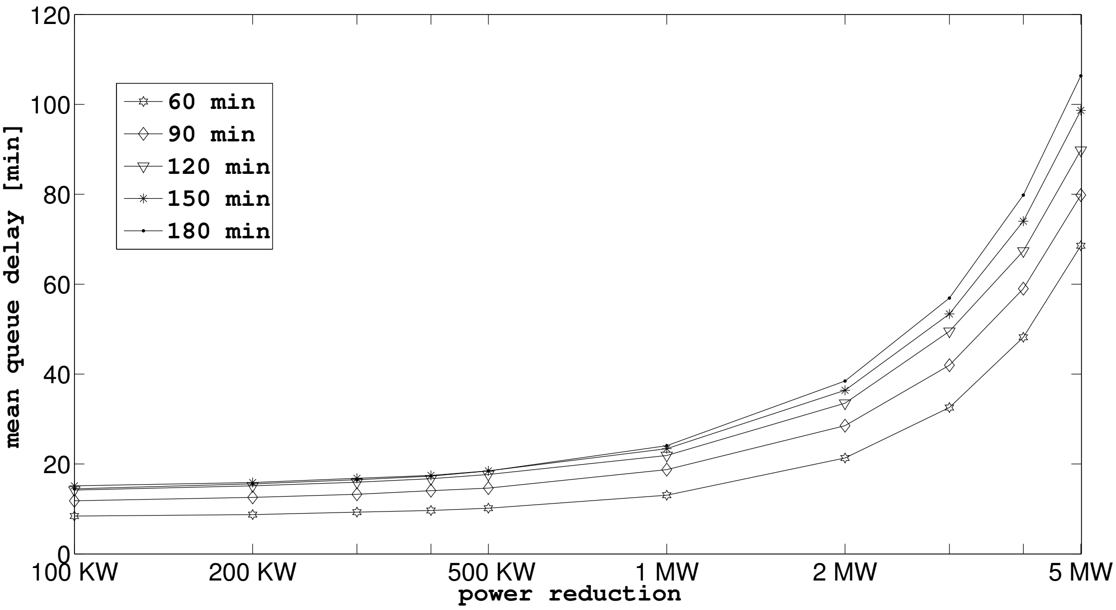


(a)

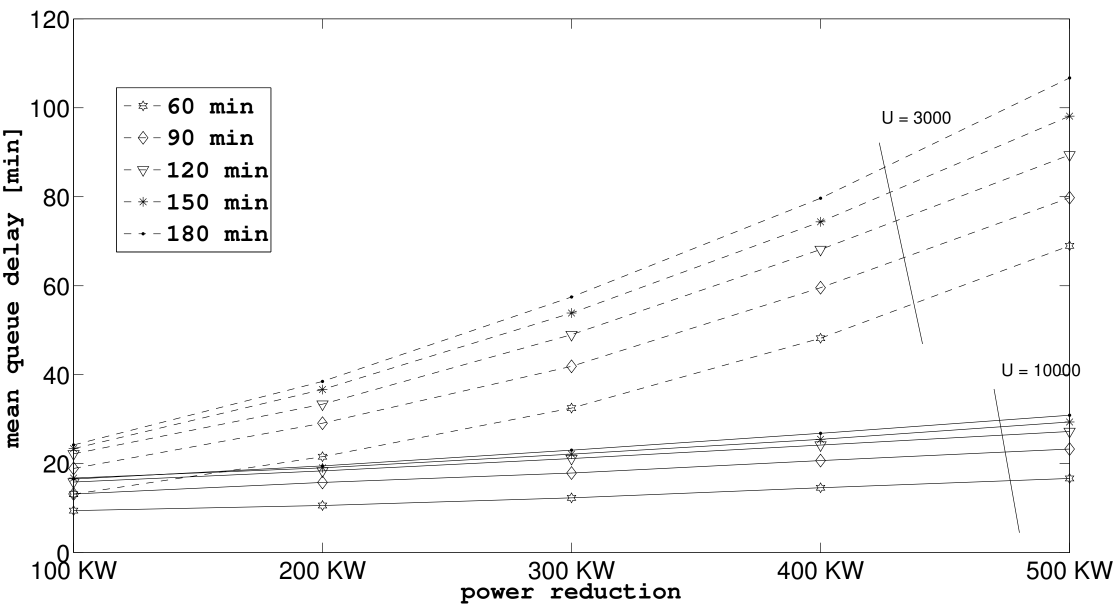


(b)

Figure 5: Average service delay for different user populations: 3000, 5000 and 10000 laundry machines.



(a)



(b)

Figure 6: Average service delay for 30000 users (a) and 3000, 5000 and 10000 users (b), under different control periods (whose legend is in (a)).

RESEARCH CENTRE
SOPHIA ANTIPOLIS – MÉDITERRANÉE

2004 route des Lucioles - BP 93
06902 Sophia Antipolis Cedex

Publisher
Inria
Domaine de Voluceau - Rocquencourt
BP 105 - 78153 Le Chesnay Cedex
inria.fr

ISSN 0249-6399